Log-Likelihood Ratio (LLR) Conversion Schemes in Orthogonal Code Hopping Multiplexing

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***Abstract*—Log-likelihood ratio (LLR) conversion schemes are proposed to reduce the effect of perforations that occur in orthogonal code hopping multiplexing (OCHM), which was previously proposed to accommodate more downlink channels than the number of orthogonal codewords. The proposed LLR conversion schemes greatly reduce the required signal-to-noise ratio (SNR) in channel decoding even when the perforation probability is high. The performance of the proposed schemes is evaluated by simulation in terms of the required E_b/N_0 for a 1% block error rate.**

***Index Terms*—Demapping, log-likelihood ratio (LLR), orthogonal code hopping multiplexing (OCHM), perforation.**

I. Introduction

INTERNET traffic in recent years has rapidly increased in a wireless domain. Bursty downlink traffic is expected to be dominant in mobile communications in the future. Low channel activity due to burstiness results in inefficient use of code channels and a shortage of available channels in the downlink. An orthogonal code hopping multiplexing (OCHM) scheme has been proposed [1], [2] to accommodate more low-activity bursty users than the number of orthogonal downlink codewords. OCHM can cause perforations among symbols, which degrade channel decoding performance when the perforation probability is high. We propose new *log-likelihood ratio (LLR) conversion* schemes that improve the decoding performance in perforation environments.

This paper is organized as follows: We briefly introduce our previously proposed OCHM scheme and LLR in Section II. New LLR conversion schemes are presented in Section III and the performance of the proposed schemes is evaluated by simulation in terms of the required E_b/N_0 for a 1% block error rate in Section IV. Conclusions are presented in Section V.

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II. Background

A. Orthogonal Code Hopping Multiplexing (OCHM)

OCHM [1], [2] was developed to accommodate more downlink orthogonal channels than the number of orthogonal codewords through statistical multiplexing. The number of dedicated orthogonal downlink channels in conventional orthogonal code-division multiplexing (OCDM)-based systems, like IS-95, cannot exceed the number of codewords regardless of downlink channel activity. It is also important to increase the use of orthogonal codewords within the maximum allowable total transmission power of the downlink in a cell. The OCHM scheme can use a mobile station (MS)-specific hopping pattern to allocate a user a code channel. Symbol hopping patterns are independent among users and occasionally cause collisions of two or more user symbols on the same code channel, which can be detected by the base station (BS) in downlink. Two or more identical symbols on the same code channel can be transmitted in the downlink. This situation yields an E_s/N_0 gain at the receiver refered to as *synergy*. However, two or more different symbols on the same code channel are perforated without transmission. *Perforation* usually maps the colliding symbol to the origin in the two-dimensional I-Q signal space. A high perforation probability degrades the performance of channel decoding.

B. Log-Likelihood Ratio (LLR)

In a soft-input decoder, channel demodulator output is generally de-mapped and used as an input value for the decoder. When we transmit d and receive y in conventional BPSK/QPSK, the LLR can be expressed as [3]

$$\begin{aligned} L(d|y) &= \log \frac{P(d=+1|y)}{P(d=-1|y)} \\ &= \log \frac{P(y|d=+1)}{P(y|d=-1)} + \log \frac{P(d=+1)}{P(d=-1)} \\ &= L(y|d) + L(d) \\ &= L(y|d), \quad \text{for equi-probable symbols} \\ &\propto y, \quad \text{for AWGN channels.} \end{aligned} \tag{1}$$

The LLR is proportional to the receiving symbol amplitude y in AWGN channels, and then, channel demodulator output is used as channel decoder input with channel reliability. These LLR calculations and conversions should be carefully examined in OCHM because perforation yields changes in symbol mapping.

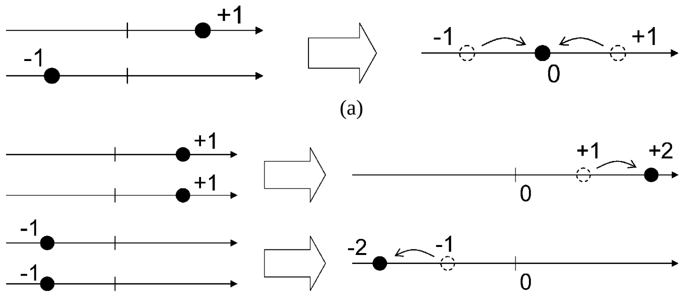

Fig. 1. Symbol mapping (BPSK). (a) Perforation. (b) Synergy with the amplitude of two symbols.

III. LLR Conversion for BPSK/QPSK

When OCHM-based downlink experiences collisions among symbols, transmitted symbols can be located at places different from the original locations in the I-Q plane due to perforations. Perforated symbols are generally placed at the origin of the plane in the case of BPSK/QPSK. According to changes in symbol mapping at the transmitter, *LLR conversion* schemes are proposed at the receiver. LLR conversion improves performance by inserting a simple conversion (demapping) function between channel demodulator output and channel decoder input.

The simplest method to implement perforation is to map the colliding symbols to the origin of the plane in data modulation. Fig. 1(a) illustrates the perforation of data symbols to the origin in BPSK. If symbols on the same codeword are identical, we can choose the amplitude of one symbol, $|\pm 1|$ [see Fig. 1(a)] or that of 2 symbols, $|\pm 2|$ [see Fig. 1(b)] as the transmitted symbol. We propose LLR conversion methods to mitigate collisions for the case illustrated in Fig. 1(a), where synergy does not affect symbol mapping[1].

When we transmit d and receive y, (1) is rewritten as (2) for equi-probable data symbols, considering perforation.

$$L(d|y) = \log \frac{(1-P_p)P(y|t=+1) + P_p \cdot P(y|t=0)}{(1-P_p)P(y|t=-1) + P_p \cdot P(y|t=0)} = \log \frac{(1-P_p)\exp\left(-\frac{(y-1)^2}{2\sigma^2}\right) + P_p \exp\left(-\frac{y^2}{2\sigma^2}\right)}{(1-P_p)\exp\left(-\frac{(y+1)^2}{2\sigma^2}\right) + P_p \exp\left(-\frac{y^2}{2\sigma^2}\right)} \quad (2)$$

where t is the transmitted symbol, P_p is the *perforation probability*, and $\sigma^2 = (1/2)(1/(E_s/N_0))$ when considering perforation. Equation (2) represents the exact calculation of LLR from y. However, the equation includes a few exponential function terms and one logarithmic function. Alternatively, we approximate (2) to a simple piecewise-linear function to reduce the computational complexity using the following approximation:

$$\sum_i A_i \exp(b_i) \approx \max_i \{A_i \exp(b_i)\}. \quad (3)$$

We divide the perforation probability P_p into two cases[2] to obtain two different conversion graphs (or functions)

$$\begin{cases} \text{case 1}: P_p & > \frac{1}{1+\exp\left(\frac{1}{2\sigma^2}\right)} \\ \text{case 2}: P_p & \leq \frac{1}{1+\exp\left(\frac{1}{2\sigma^2}\right)} \end{cases}. \quad (4)$$

[1]The same procedure can be applied to the case including Fig. 1(b).

[2]Cases and region boundaries are determined by the shape of conversion graphs and the point where competing exponential functions become equal.

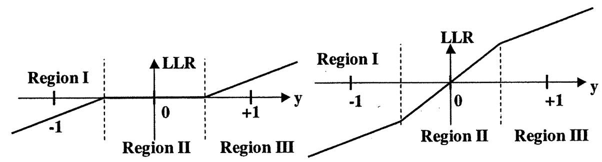

Fig. 2. Linear LLR conversion in case of considering perforation.

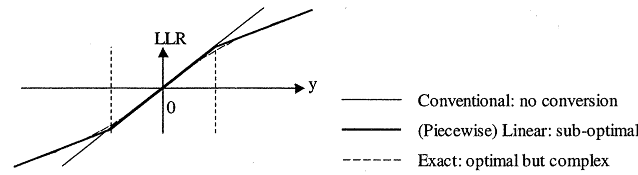

Fig. 3. LLR conversion schemes in case of considering perforation.

Equation (5) is the approximated LLR for case 1.

$$L(d|y) \approx \begin{cases} \log \frac{P_p}{1-P_p} + \frac{1}{\sigma^2}\left(y + \frac{1}{2}\right), & \text{Region I} \\ 0, & \text{Region II} \\ \log \frac{1-P_p}{P_p} + \frac{1}{\sigma^2}\left(y - \frac{1}{2}\right), & \text{Region III.} \end{cases} \quad (5)$$

The approximated LLR is a piecewise-linear first-order equation of y. Approximated LLR conversion is possible when the receiver knows the P_p value. However, estimation of P_p requires consideration as another issue. Similarly, LLR is approximated for case 2.

$$L(d|y) \approx \begin{cases} \log \frac{P_p}{1-P_p} + \frac{1}{\sigma^2}\left(y + \frac{1}{2}\right), & \text{Region I} \\ \frac{2y}{\sigma^2}, & \text{Region II} \\ \log \frac{1-P_p}{P_p} + \frac{1}{\sigma^2}\left(y - \frac{1}{2}\right), & \text{Region III.} \end{cases} \quad (6)$$

Fig. 2 illustrates two cases of piecewise-linear LLR conversion. It is preferable to erase symbols near the origin for a high perforation probability in Fig. 2(a). The slope in Region II of Fig. 2(b) is the same as the conventional scheme. Region II extends as the perforation probability decreases.

In summary, we propose an exact LLR conversion method [see (2)] and derive a piecewise-linear LLR conversion method [see (5) and (6)] by approximation in order to reduce the computational complexity. Fig. 3 illustrates the three LLR conversion schemes. The piecewise-linear scheme exhibits a performance similar to the exact conversion scheme.

IV. Numerical Results

Simulation environments are described assuming perfect channel estimation, equal gain combining, and BPSK/QPSK. Wireless channels are assumed to experience either AWGN or independent (uncorrelated) Rayleigh fading. No specific code hopping patterns are designated and symbol perforations occur randomly. Turbo coding in 3GPP specifications is considered with the decoder using the soft output Viterbi algorithm (SOVA) [4]. A 1% BLER (Block Error Rate) [5] is assumed as a performance measure. The size of an encoder block is 1000 bits and the number of encoder blocks used in simulation is 10 000. The code rate r is 1/4.

Fig. 4 illustrates the required E_b/N_0 for a 1% BLER versus the perforation probability for the three LLR conversion schemes. Fig. 4(a) and (b) show the performance on AWGN

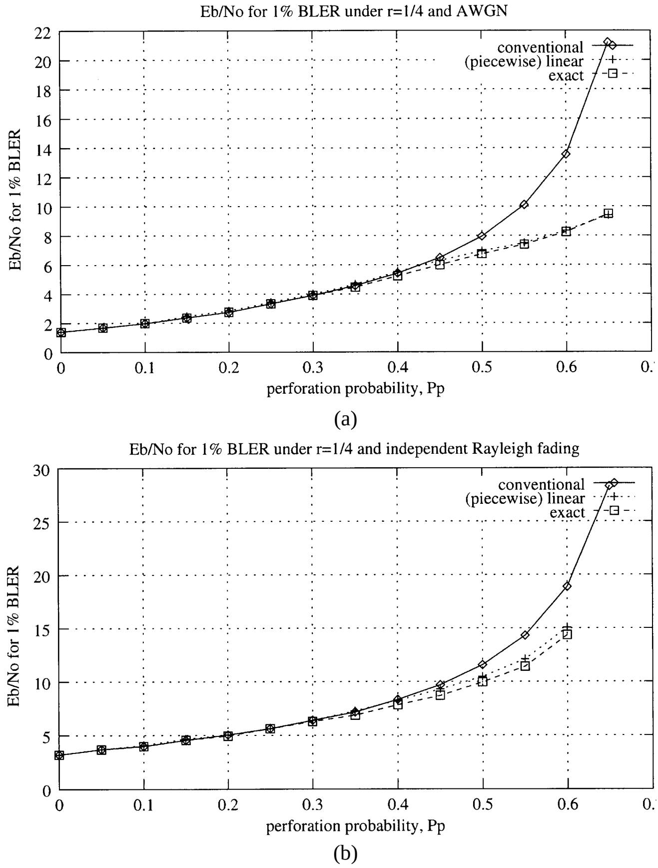

Fig. 4. Required E_b/N_0 for a 1% block error rate. (a) AWGN channels. (b) Independent Rayleigh fading channels.

channels and independent Rayleigh fading channels, respectively. AWGN channels exhibit slowly varying fading channels while independent Rayleigh fading channels show rapidly varying fading. The three schemes yield similar performances when P_p is low. However, the two proposed schemes reduce the required E_b/N_0 by up to 10 dB when P_p is high. The small performance difference between the exact and the proposed piecewise-linear approximation scheme is within 0.3 dB. It is appropriate to adopt the piecewise-linear LLR conversion scheme for $P_p > 0.3$ and $r = 1/4$. When the maximum receivable E_b/N_0 is limited, the proposed schemes allow a larger range of P_p. As an example, the proposed schemes extend the acceptable P_p from 0.5 to 0.6 for an E_b/N_0 value of 8 dB in AWGN channels. This results in an increase in the allowable number of users from 887 to 1173 [6] for 64 code channels[3] with an activity factor of 0.1.

The required E_b/N_0 values of $8 \sim 10$ dB for large values of P_p are high in the current interference-limited[4] system. However, they can be supportable in future advanced code-limited systems. Interference from other users limits the maximum receivable E_b/N_0 (or E_b/I_0) at the MS, which should be larger than the required E_b/N_0. Low activity data traffic (Internet) will result in a decrease in interference and an increase in the maximum receivable E_b/N_0. Interference can increase as more users are accommodated through statistical multiplexing of low activity channels.

Another consideration for future wireless systems is adaptive arrays. A multibeam antenna with M beams reduces the number of interferers by a factor of M [7], and adaptive arrays provide limited additional interference suppression. As an example, a BS antenna with 8 beams provides interference suppression of 9 dB both on the downlink and the uplink, which increases the maximum receivable E_b/N_0 by 9 dB. If large values of P_p become acceptable, the system can accommodate many more users. An alternative approach for a high E_b/N_0 value is to increase the target BLER from 1% to $5 \sim 10\%$, which will decrease the required E_b/N_0 but may cause an increase in delay and a decrease in throughput due to retransmissions.

[3]OCHM can accommodate significantly more downlink channels than the number of orthogonal codewords, 64 [1].

[4]An interference(power)-limited situation means that the maximum allowable number of users is less than the number of code channels and the number is limited by interference. The code-limited system here indicates that users fully utilize the number of code channels and the number of users can exceed the number of code channels if a suitable scheme (OCHM) is applied.

V. Conclusions

An orthogonal code hopping multiplexing (OCHM) scheme has been proposed [1], [2] as a novel statistical multiplexing scheme for orthogonal downlink to accommodate more low-activity bursty users than the number of orthogonal downlink codewords. OCHM can cause perforations among symbols, which degrade the performance of channel decoding when the perforation probability is high. We propose new LLR conversion schemes that improve the decoding performance in perforation environments. The schemes insert a simple conversion function between channel demodulator output and channel decoder input. We propose two types of LLR conversion function. The proposed schemes reduce the required E_b/N_0 by up to 10 dB when P_p is high. The piecewise-linear (sub-optimum) conversion scheme yields performance similar to the exact (optimum) scheme, and reduces the computational complexity.

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